SOLUTION OF SHORT QUESTIONS

Short Questions

Write the short answers of the following Questions:

Expand by Binomial theorem, Q. No.1 to 4.

Q.1: $(2x-3y)^4$ **(IA-2016)**

Sol. $(2x-3y)^4$

$$=\binom{4}{0}(2x)^4(3y)^0-\binom{4}{1}(2x)^3(3y)+\binom{4}{2}(2x)^2(3y)^2-\binom{4}{3}(2x)^1(3y)^3+\binom{4}{4}(2x)^0(3y)^4$$

$$=(1)(16x^4)(1)-4(8x^3)(3y)+6(4x^2)(9y^2)-4(2x)(27y^3)+(1)(1)(81y^4)$$

$$=\boxed{16x^4-96x^3y+216x^2y^2-216xy^3+81y^4}$$

Q.2: $\left(\frac{x}{y}+\frac{y}{x}\right)^4$ **(IIA-2016), (IA-2017), (IIA-2018) (IIA-2021)**

Sol. $\left(\frac{x}{y}+\frac{y}{x}\right)^4$

$$=\binom{4}{0}\left(\frac{x}{y}\right)^4\left(\frac{y}{x}\right)^0+\binom{4}{1}\left(\frac{x}{y}\right)^3\left(\frac{y}{x}\right)^1+\binom{4}{2}\left(\frac{x}{y}\right)^2\left(\frac{y}{x}\right)^2+\binom{4}{3}\left(\frac{x}{y}\right)^1\left(\frac{y}{x}\right)^3+\binom{4}{4}\left(\frac{x}{y}\right)^0\left(\frac{y}{x}\right)^4$$

$$=(1)\left(\frac{x^4}{y^4}\right)(1)+4\left(\frac{x^3}{y^3}\right)\left(\frac{y}{x}\right)+6\left(\frac{x^2}{y^2}\right)\left(\frac{y^2}{x^2}\right)+4\left(\frac{x}{y}\right)\left(\frac{y^3}{x^3}\right)+1(1)\frac{y^4}{x^4}$$

$$=\boxed{\frac{x^4}{y^4}+4\frac{x^2}{y^2}+6+4\frac{y^2}{x^2}+\frac{y^4}{x^4}}$$

Q.3: $\left(\frac{x}{2}-\frac{2}{y}\right)^4$ **(IA-2019)**

Sol. $\left(\frac{x}{2}-\frac{2}{y}\right)^4$

$$=\binom{4}{0}\left(\frac{x}{2}\right)^4\left(\frac{2}{y}\right)^0-\binom{4}{1}\left(\frac{x}{2}\right)^3\left(\frac{2}{y}\right)^1+\binom{4}{2}\left(\frac{x}{2}\right)^2\left(\frac{2}{y}\right)^2-\binom{4}{3}\left(\frac{x}{2}\right)^1\left(\frac{2}{y}\right)^3+\binom{4}{4}\left(\frac{x}{2}\right)^0\left(\frac{2}{y}\right)^4$$

SOLUTION OF SHORT QUESTIONS

$$=(1)\left(\frac{x^4}{16}\right)(1)-4\left(\frac{x^3}{8}\right)\left(\frac{2}{y}\right)+6\left(\frac{x^2}{4}\right)\left(\frac{4}{y^2}\right)-4\left(\frac{x}{2}\right)\left(\frac{8}{y^3}\right)+(1)(1)\left(\frac{16}{y^4}\right)$$

$$=\boxed{\frac{x^4}{16}-\frac{x^3}{y}+6\frac{x^2}{y^2}-16\frac{x}{y^3}+\frac{16}{y^4}}$$

Q.4: $\left(x+\frac{1}{x}\right)^4$ (IIA-2017), (IIA-2020), (IA-2021)

Sol. $\left(x+\frac{1}{x}\right)^4$

$$=\binom{4}{0}x^4\left(\frac{1}{x}\right)^0+\binom{4}{1}x^3\left(\frac{1}{x}\right)^1+\binom{4}{2}x^2\left(\frac{1}{x}\right)^2+\binom{4}{3}x^1\left(\frac{1}{x}\right)^3+\binom{4}{4}x^0\left(\frac{1}{x}\right)^4$$

$$=(1)x^4(1)+4x^3.\frac{1}{x}+6x^2.\frac{1}{x^2}+4x.\frac{1}{x^3}+(1)(1)\frac{1}{x^4}$$

$$=\boxed{x^4+4x^2+6+\frac{4}{x^2}+\frac{1}{x^4}}$$

Q.5: State Binomial Theorem for positive integer n.

(IIA-2020), (IA-2021)

Ans. The rule for expansion of $(a+b)^n$, where 'n' is any positive integral power, is called binomial theorem, and defined as:

$$(a+b)^n=\binom{n}{0}a^nb^0+\binom{n}{1}a^{n-1}b^1+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{n}a^0b^n$$

Calculate the following by Binomial Theorem up to two Decimal places.

Q.6: $(1.02)^{10}$ (IIA-2018), (IIA-2022)

Sol. $(1.02)^{10}=(1+0.02)^{10}$

$$=\binom{10}{0}(1)^{10}(0.02)^0+\binom{10}{1}(1)^9(0.02)^1+\binom{10}{2}(1)^8(0.02)^2+\ldots$$

$$=(1)(1)(1)+10(1)(0.02)+45(1)(0.0004)+\ldots$$

$$=1+0.2+0.018+\ldots=1.2180=\boxed{1.22}$$

SOLUTION OF SHORT QUESTIONS

Q.7: $(1.04)^5$ (IA-2016), (IA-2019), (IIA-2019), (IA-2021)

Sol. $(1.04)^5 = (1+0.04)^5$

$$= \binom{5}{0}(1)^5(0.04)^0 + \binom{5}{1}(1)^4(0.04)^1 + \binom{5}{2}(1)^3(0.04)^2 + \dots$$

$$= (1)(1)(1) + 5(1)(0.04) + 10(1)(0.0016) + \dots$$

$$= 1 + 0.2 + 0.016 + \dots = 1.2160 = \boxed{1.22}$$

Q.8: Find the 7th term in the expansion of $\left(x - \frac{1}{x}\right)^9$

(IIA-2016), (IA-2017), (IA-2018), (IIA-2019), (IA-2021), (IA-2022)

Sol. Here $a = x$, $b = -\frac{1}{x}$, $n = 9$, $r = 6$

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$T_{6+1} = \binom{9}{6} (x)^{9-6} \left(-\frac{1}{x}\right)^6$$

$$T_7 = 84x^3\left(\frac{1}{x^6}\right)$$

$$\boxed{T_7 = \frac{84}{x^3}}$$

Q.9: Find the 6th term in the expansion of $(x + 3y)^{10}$.

(IIA-2018)

Sol. Here $a = x$, $b = 3y$, $n = 10$, $r = 5$

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$T_{5+1} = \binom{10}{5} (x)^{10-5} (3y)^5$$

$$T_6 = (252)(x^5)(243y^5)$$

$$\boxed{T_6 = 61236x^5y^5}$$

Q.10: Find 5th term in the expansion of $\left(2x - \frac{x^2}{4}\right)^7$

(IA-2016), (IIA-2017), (IIA-2020)

Sol. Here $a = 2x$, $b = -\frac{x^2}{4}$, $n = 7$, $r = 4$

SOLUTION OF SHORT QUESTIONS

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$T_{4+1} = \binom{7}{4} (2x)^{7-4} \left(-\frac{x^2}{4}\right)^4$$

$$T_5 = (35)(8x^3)\left(\frac{x^8}{256}\right)$$

$$\boxed{T_5 = \frac{35}{32} x^{11}}$$

Expand up to three terms:

Q.11: $(1 + 2x)^{-2}$ (IA-2016), (IIA-2016), (IA-2017), (IA-2019)

Sol. $(1+2x)^{-2}$

Put $b = 2x$ & $n = -2$ in Binomial series Formula, we have

$$= 1 + (-2)(2x) + \frac{(-2)(-2-1)}{2!}(2x)^2 + \ldots$$

$$= 1 - 4x + \frac{(-2)(-3)}{2}(4x^2) + \ldots = \boxed{1 - 4x + 12x^2 + \ldots}$$

Q.12: $\dfrac{1}{(1+x)^2}$ (IIA-2020)

Sol. $\dfrac{1}{(1+x)^2} = (1+x)^{-2}$

Put $b = x$ & $n = -2$ in Binomial series Formula, we have

$$= 1 + (-2)(x) + \frac{(-2)(-2-1)}{2!}(x)^2 + \ldots$$

$$= 1 - 2x + \frac{(-2)(-3)}{2}x^2 + \ldots = \boxed{1 - 2x + 3x^2 + \ldots}$$

Q.13: $\dfrac{1}{\sqrt{1+x}}$ (IIA-2018), (IIA-2019), (IA-2022)

Sol. $\dfrac{1}{\sqrt{1+x}} = (1+x)^{-1/2}$

Put $b = x$ & $n = -1/2$ in Binomial series Formula, we have

SOLUTION OF SHORT QUESTIONS

$$=1+\left(-\frac{1}{2}\right)(x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(x)^2+\ldots$$

$$=1-\frac{x}{2}+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)x^2+\ldots=\boxed{1-\frac{x}{2}+\frac{3}{8}x^2+\ldots}$$

Q.14: $(4-3x)^{1/2}$ (IA-2018)

Sol. $(4-3x)^{1/2}=\left[4\left(1-\frac{3x}{4}\right)\right]^{1/2}=2\left(1-\frac{3x}{4}\right)^{1/2}$

Put $b=-\frac{3x}{4}$ & $n=\frac{1}{2}$ in Binomial series Formula, we have

$$=2\left[1+\left(\frac{1}{2}\right)\left(-\frac{3x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}\left(-\frac{3x}{4}\right)^2+\ldots\right]$$

$$=2\left[1-\frac{3x}{8}+\frac{1}{2}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{9x^2}{16}\right)+\ldots\right]$$

$$=2\left[1-\frac{3x}{8}-\frac{9x^2}{128}+\ldots\right]=\boxed{2-\frac{3x}{4}-\frac{9x^2}{64}+\ldots}$$

Q.15: Using the Binomial series, calculate $\sqrt[3]{65}$ to the nearest hundredth. (IIA-2016)

Sol. $\sqrt[3]{65}=(64+1)^{1/3}=\left[64\left(1+\frac{1}{64}\right)\right]^{1/3}=4\left(1+\frac{1}{64}\right)^{1/3}$

$$=4\left[1+\left(\frac{1}{3}\right)\left(\frac{1}{64}\right)+\ldots..\right]=4\left[1+\frac{1}{192}+\ldots\right]=4(1.0052)=\boxed{4.02}$$

Which will be the middle term/terms in the expansion:

Q.16: $(2x+3)^{12}$ (IIA-2016)

Sol. Here $a=2x$, $b=3$, $n=12$

SOLUTION OF SHORT QUESTIONS

As $n = 12$ (Even), so

$$\text{Middle term} = \left(\frac{n}{2}+1\right)^{th} = \left(\frac{12}{2}+1\right)^{th} = (6+1)^{th} = 7^{th}\text{ term}$$

$\boxed{\text{Hence } T_7 \text{ is a middle term}}$

Q.17: $\left(x+\frac{3}{x}\right)^{15}$ (IA-2016), (IA-2017), (IA-2019)

Sol. Here $a = x,\ b = \frac{3}{x},\ n = 15$

As $n = 15$ (Odd), so

$$\text{Middle term} = \left(\frac{n+1}{2}\right)^{th} + \left(\frac{n+3}{2}\right)^{th} \text{ terms.}$$

$$= \left(\frac{15+1}{2}\right)^{th} + \left(\frac{15+3}{2}\right)^{th} \text{ terms.}$$

$$= 8^{th} + 9^{th} \text{ terms.}$$

$\boxed{\text{Hence } T_8 \ \& \ T_9 \text{ are two middle terms}}$

Q.18: Which term is the middle term/terms in the Binomial expansion of $(a+b)^n$. (IIA-2017), (IA-2018)

Sol. (i) <u>**When n is even**</u>

Then, Middle term $= \left(\frac{n+2}{2}\right)^{th}$ term

Sol. (ii) <u>**When n is odd**</u> (IIA-2020), (IA-2021)

Then, there are two middle terms:

$$\text{Middle term} = \left(\frac{n+1}{2}\right)^{th} + \left(\frac{n+3}{2}\right)^{th} \text{ terms.}$$